PROCESS CHARACTERISTICS OF SCREW IMPELLERS WITH A DRAUGHT TUBE FOR NEWTONIAN LIQUIDS. PUMPING CAPACITY OF THE IMPELLER

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Velocity profiles and pumping capacity have been determined using a thermistor anemometer in a vessel equipped with a screw impeller. In region of the creeping flow of a Newtonian liquid, *i.e.* for Re < 15, the dimensionless pumping capacity is dependent on the geometrical arrangement of the mixing system. The efficiency was assessed of individual configuration from the value of energy criterion expressing the dimensionless power requirements for recirculation of a highly viscous liquid in a vessel equipped with a screw impeller.

Rotation of a screw impeller in an immobile draught tube gives rise to circulatory flows from the screw *via* the annular space between the cylinders back into the screws. This flow is characterized by the volume pumping capacity $\dot{V}$ of the impeller. At the same time it is necessary to overcome the pressure drop induced by fraction of the liquid – a highly viscous and a Newtonian one – on the walls, the bottom and the draught tube.

The pumping capacity of a screw impeller with a draught tube is obviously dependent, similarly as its power input[1], on the shape of the screw and the pressure loss in the recirculation loop. This question has been studied on screw impellers earlier by some authors[2–8]. The aim of this work is to assess the effect of some geometrical simplexes of the mixing system on the magnitude of the power input in the creeping flow region (*i.e.* at very low values of the Reynolds number).

The pumping capacity of a screw impeller could be theoretically determined from the knowledge of the velocity field in the mixed highly viscous batch by solving the Navier–Stokes equation. For this system though this task is so complex that it is practically unfeasible.

For the volume flow rate through the annular space one can use the following relation

$$\dot{V} = 2\pi \int_{D'/2}^{D/2} v_z r \, \mathrm{d}r \qquad (1)$$

This flow rate is identical with the flow rate through the draught tube, *i.e.* with the pumping capacity of the screw impeller.

By introducing the dimensionless quantities $r^* = r/d$ and $v_z^* = v_z/nd$ Eq. (*1*) may be rearranged into the form

$$\dot{V}/nd^3 = 2\pi \int_{D'/2d}^{D/2d} v_z^* r^* \, dr^* . \qquad (2)$$

The left hand side of Eq. (*1*) represents the dimensionless pumping capacity which in the following shall be termed the flow rate criterion *Kv*

$$Kv = \dot{V}/nd^3 . \qquad (3)$$

The flow rate criterion, Kv, according to Eq. (*2*) depends on the magnitude of the dimensionless axial velocity, v_z^*, which for low Reynolds numbers is a function of the geometrical configuration of the mixing system only[9–11]. It may be therefore expected that in the creeping flow regime, Kv is a constant for geometrically similar systems.

Based on the analogy between the mixing and the extruding screws it may be concluded that the pumping capacity of the given screw impeller should depend on the magnitude of the pressure loss in the recirculation loop. This assumption has been verified theoretically as well as experimentally by Rieger[2]. Similarly as the extruding screw[9], the screw impeller exhibits also a linear pumping characteristic[2] (Fig. 1).

In dimensionless form this characteristic for the creeping flow region may be expressed by[2]

$$\frac{\dot{V}}{nd^3} = Kv_{max} - k\frac{\Delta p}{\mu n} . \qquad (4)$$

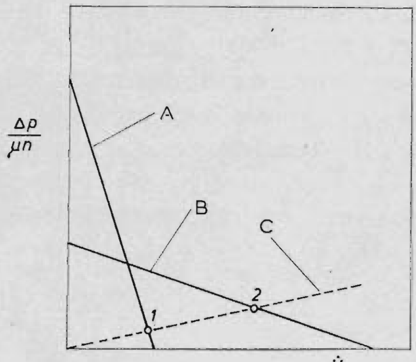

FIG. 1

Dimensionless pumping characteristics of screws. A Low lead screw; B high lead screw; C extruding characteristic (recirculation), *1*, *2* operating points of the screws

Here Kv_{max} represents the drag flow, *i.e.* the pumping capacity at zero pressure drop. The second term on the right hand side of Eq. (*4*) expresses the magnitude of the pressure (reverse) flow. The magnitude of Kv_{max} depends on the shape of the screw and the jacket[9] (or the draught tube). The same is sure also about the value of k. The magnitude of the dimensionless pressure loss, Kt

$$\mathrm{Kt} = \Delta p / \mu n \tag{5}$$

depends on the geometrical arrangement of the whole system.

Based on the above analysis it may be expected that in the laminar flow region the flow rate criterion, Kv, shall depend on the geometrical simplexes of the mixing system

$$\mathrm{Kv} = f\left(\frac{D}{d}, \frac{s}{d}, \frac{d_0}{d}, \frac{h_v}{d} \ldots\right). \tag{6}$$

Several methods have been described in the literature to measure the pumping capacity of a screw impeller. The most frequently used method utilizes measurement of the volumetric flow of liquid driven by the screw in a continuous-flow set-up[2-5]. From the practical geometrical configurations this arrangement differs in the absence of the recirculatory flow of the highly viscous liquid. This yields though the pumping capacity of the screw[2] (throttling the exit stream).

The method based on tracing the circulation of solid particles suspended in liquid, the so-called "flow follower"[6,7], starts from the assumption that the mean residence time of the particle is identical with the recirculation time in the vessel.

Another possibility is the interpretation of records of the course of homogenation of the mixed batch[8,10], where the recirculation of the batch reflects in the oscillatory behaviour of concentration in a given position of the mixed system.

The results of the above cited papers have shown the value of the flow rate criterion, Kv, in the creeping flow region, *i.e.* roughly for $\mathrm{Re} < 20$, to depend solely on the geometrical configuration of the mixing system (or also on the value of the dimensionless pressure loss, Kt).

The knowledge of the pumping capacity of the screw impeller, together with its power input, may serve as an information for the evaluating of individual geometrical configurations from the view point of energy consumption. For this purpose one can start from the criterion proposed by Hoogendoorn[12-15]

$$E_{\mathrm{H}}^{*} \equiv \frac{P\tau_{\mathrm{H}}^{2}}{\mu D^{3}} = \mathrm{Po\ Re\ Ho}^{2}\left(\frac{d}{D}\right)^{3}. \tag{7}$$

Here the time of homogenation may be replaced by the mean recirculation time of the highly viscous batch in the vessel

$$\tau_c = V/\dot{V} \tag{8}$$

$$\tau_c \sim \frac{1}{\mathrm{Kv}\, n}\left(\frac{D}{d}\right)^3 . \tag{9}$$

The modified energy criterion may then take the following form

$$E_c^* \equiv \frac{P\tau_c^2}{\mu D^3} = \mathrm{Po\ Re\ Kv}^{-2}\left(\frac{D}{d}\right)^3 . \tag{10}$$

Because the product (Po Re) and Kv depend in the creeping flow region on the geometrical configuration only, the criterion E_c^* shall depend also on the geometry only.

EXPERIMENTAL

The pumping capacity of the screw impellers with draught tubes was assessed on the basis of measurement of the volume flow rate, $\dot{V}$, in the annular space. The relationship used for the evaluation of $\dot{V}$ was Eq. (1). The velocity profile $v_z(r)$ was detected by a thermistor anemometer.

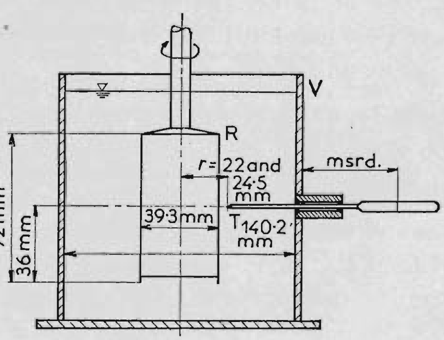

FIG. 2

Method of calibration of the thermistor anemometer. R Rotating cylinder, V calibrating vessel, T thermistor probe

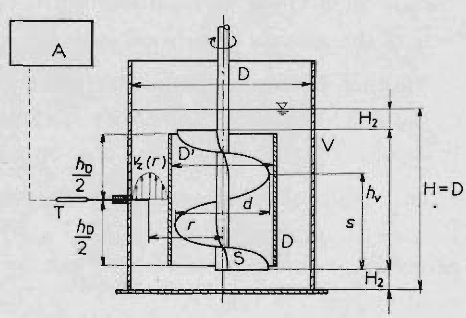

FIG. 3

Experimental arrangement of the mixing system for velocity measurement in an annular space and a high viscosity batch. S Screw impeller, D draught tube, V mixing vessel, T thermistor probe, A measuring instrument of the anemometer

The principle of its function, the wiring of the anemometer and the temperature bridge as well as the analysis of operating conditions have been described elsewhere[12].

The anemometric function of the bead glass-covered thermistor (heated by a constant current) was based on detecting the change of electric resistance at constant current at — zeroed resistance of the bridge: (R_0), under the conditions of free convection of the highly viscous liquid surrounding the probe — the measurements proper: ($R_0 + \Delta R$), under the conditions of the flow around the probe.

Note: This value of ΔR was always positive in view of the fact that the resistance of the thermistor increases with decreasing temperature.

At the same time it was possible, by switching the instrument, to check the temperature in the measuring point (not heated thermistor).

The measuring probe was manufactured from a stainless steel tube, 2·5 mm in diameter, 115 mm long mounted in a clamp. The tip of the probe was formed by the bead of the glass covered thermistor (13 NR 09/A type, made by Pramet), cemented into the tube by epoxy resin. The extreme tip of the probe was the bead of the thermistor about 1·2 mm in diameter smoothly changing into a cone of the apex angle 30°.

Calibration of the probe. The system used to calibrate the anemometer consisted of a cylindrical calibration vessel 140·2 mm in internal diameter and a cylinder of a rotational viscometer, Rheotest 2, 39·3 mm (Fig. 2) in diameter, rotating coaxially with the former cylinder.

The probe was placed radially into the immediate proximity (2·5 mm,) partly also 5 mm) of the rotating cylinder.

The velocity of the flow of the liquid past the thermistor bead was computed from the relation[13]

$$v_\varphi = \frac{2\pi n}{\lambda^2 - 1} \left(\frac{r_2^2 - r^2}{r} \right), \tag{11}$$

where n stands for the frequency of revolution of the rotating cylinder, λ is the ratio of diameters of the cylinders (here $\lambda = 3{\cdot}58$), r_2 is the radius of the outer cylinder ($r_2 = 70{\cdot}1$ mm) Using 24 different values of the frequency of revolution of the rotational cylinder, various velocities past the probe could be adjusted in the limits $v_\varphi \in \langle 0{\cdot}49 \,.\, 10^{-3}; 0{\cdot}24 \rangle$ m s^{-1}.

The calibration curve $\Delta R = f(v_\varphi)$ is valid for only the given liquid at constant temperature. Accordingly, the calibration of the anemometer was carried out always with the sample of the liquid taken from the mixing vessel prior to and after the measurement (identical temperatures and R_0 in the calibration and the mixing vessel).

The measurement of the velocity profile in the mixing vessel. The principle of measurement of axial velocity of the flow in the annular space is apparent from Fig. 3. The thermistor anemometer was placed radially in the mixing vessel half-way of its depth (as well as the depth of the draught cylinder). This enabled measurements to be carried out in the range $r \in \langle D/2; D'/2 \rangle$. Position of thermistor in the vessel (r) was determined according to retracting the probe from the vessel. For each position of the probe the value of ΔR was determined. The calibration curve then served to read off corresponding value of velocities past the probe. At the same time the temperature of the mixed batch was checked. The maximum temperature difference across the channel cross section amounted to 0·2 K. The evaluation of the volume flow rate was carried out by graphical integration of the velocity profile.

The measurements were carried out in a vessel 290 mm in diameter with a flat bottom. The screw impellers with draught tubes (D'/d — 1·1) were located in the axis of the vessel; principal dimensions are summarized in Table I.

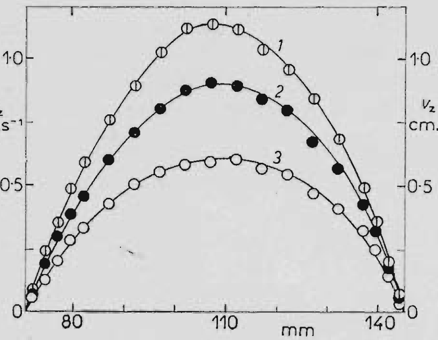

FIG. 4

Velocity profile in a highly viscous batch in the annular space of the screw impeller No 2C, $s/d = 0{\cdot}60$ $D/d = 2{\cdot}30$. *1* $n = 59\ \text{min}^{-1}$ Re = 1·52 Kv = 0·193; *2* $n = 49\ \text{min}^{-1}$ Re = 1·26 Kv = 0·188; *3* $n = 35{\cdot}5\ \text{min}^{-1}$ Re = 0·92 Kv = 0·179

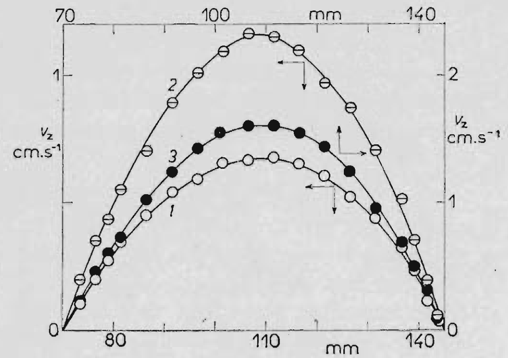

FIG. 5

Velocity profile in a highly viscous batch in the annular space of the screw impeller No 2E, $s/d = 1{\cdot}00 D/d = 2{\cdot}30$. *1* $n = 28\ \text{min}^{-1}$ Re = 7·44 Kv = 0·244, *2* $n = 42\ \text{min}^{-1}$ Re = 11·16 Kv = 0·270; *3* $n = 51{\cdot}5\ \text{min}^{-1}$ Re = 13·69 Kv = 0·307

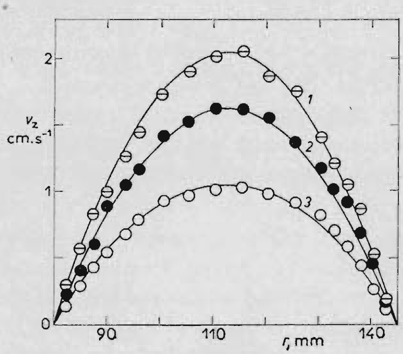

FIG. 6

Velocity profile in a highly viscous batch in the annular space of the screw impeller No 3, $s/d = 1{\cdot}00 D/d = 2{\cdot}00$. *3* $n = 23\ \text{min}^{-1}$ Re = 2·77 Kv = 0·281; *2* $n = 35\ \text{min}^{-1}$ Re = 4·21 Kv = 0·279; *1* $n = 40{\cdot}5\ \text{min}^{-1}$ Re = 4·87 Kv = 0·277

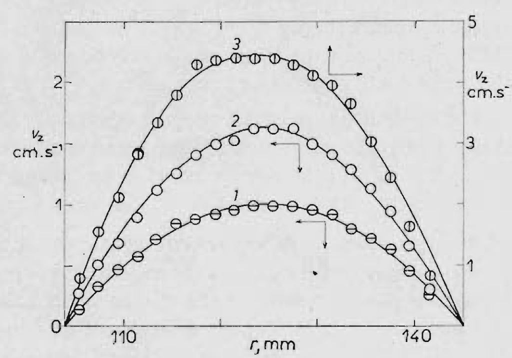

FIG. 7

Velocity profile in a highly viscous batch in the annular space of the screw impeller No 4, $s/d = 0{\cdot}93 D/d = 1{\cdot}59$. *1* $n = 16\ \text{min}^{-1}$ Re = 1·64 Kv = 0·130; 2 $n = 21\ \text{min}^{-1}$ Re = 2·15 Kv = 0·157; *3* $n = 67\ \text{min}^{-1}$ Re = 6·87 Kv = 0·140

The shaft of the impeller was driven by a hydromotor JHMA 2 with a continuous control of the frequency of revolution. The whole set-up, including the method of measurement of rpm, has been described in the previous paper[1,12]. For liquids to be mixed we selected several Newtonian starch syrup water solutions. Their viscosity and density were measured at the temperature of experiment (and calibration) using a Rheotest RV viscometer and a pycnometer.

TABLE I

Dimensions of screw impellers, mm (d_0 = 32 mm, D'/d = 1·1)

Impeller	1	2A	2B	2C	2D	2E	2F	2G	3	4
d	86	126	126	126	126	126	126	126	145	183
s	86	42	58	76	94·5	126	168	189	145	170
h_v	129	189	189	189	189	189	189	189	218	250

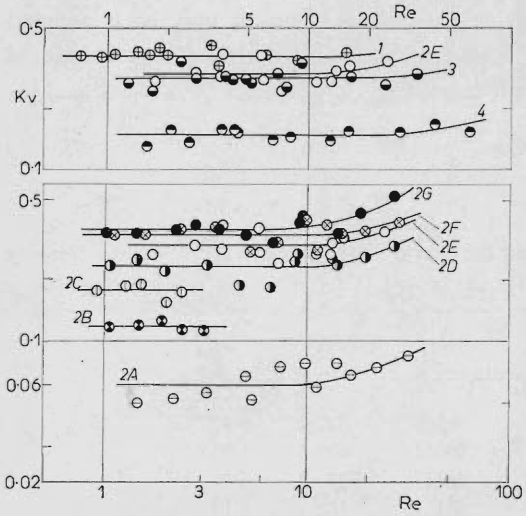

FIG. 8

Dependence of the criterion Kv on the Reynolds number, Re. Designation of curves corresponds to that of the impellers in Table I: *1* (⊕); *2A* (⊖); *2B* (⊗); *2C* (⦶); *2D* (◑); *2E* (○); *2F* (⊗); *2G* (●); *3* (◒); *4* (◓)

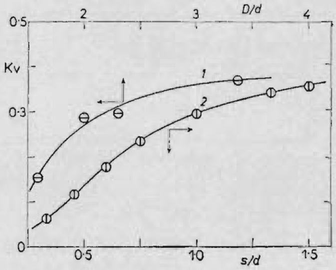

FIG. 9

The effect of the simplexes of geometrical similarity on the flow rate criterion under the creeping flow regime. *1* Kv = $f(D/d)$ for s/d = 1·0; *2* Kv = $f(s/d)$ for D/d = 2·3

RESULTS

The method described in the previous part yielded velocity profiles in the space between the wall of the cylinder and the draught tube during mixing of highly viscous liquids. The experimental runs covered changes of geometry of the system, given by the replacement of the impeller (Table I), and in addition, changes due to different physical properties of liquids and the frequency of revolution of the impeller.

Examples of several velocity profiles $v_z(r)$ for certain geometrical configurations are presented in Figs 4–7. These dependences were in all cases (110 altogether) close to a symmetric distribution with the maximum velocity approximately half way of the width of the channel, similarly as found by Hoogendoorn and coworkers[15].

The curves $v_z(r)$ served to evaluate the magnitude of the volume flow rate through the annulus, *i.e.* the pumping capacity of the screw impeller, $\dot{V}$.

Eq. (*3*) was then used to calculate the magnitude of the flow rate criterion, Kv, Fig. 8 shows the dependence Kv $= f(\text{Re})$ with the simplex of the relative lead of the screw $-s/d$ (keeping D/d constant) as a parameter. Here presented results suggest that Kv increases with increasing value of the simplex s/d. The same Fig. 8 shows also the dependence Kv $= f(\text{Re})$ with the parameter D/d, keeping s/d constant. It turns out that the smaller the impeller in the given vessel, the greater the value of Kv. At the same time it may be observed for all configurations that for Re > 15 the value of the flow rate criterion begins to increase. Thus it may be concluded that the creeping flow region (*i.e.* the region of negligible inertia forces compared to the viscous forces) is fixed by the conditions Re < 15.

TABLE II

Mean values of the flow rate criterion $\overline{\text{Kv}}$ and the energy criterion E_c^*, Eq. (*10*) for the creeping flow regime (Re < 15)

Impeller	D/d	s/d	$\overline{\text{Kv}}$	PoRe[1]	E_c^*
1	3·37	1·00	0·368	207·2	$5{\cdot}85 \,.\, 10^4$
2A	2·30	0·33	0·061	473·6	$1{\cdot}53 \,.\, 10^6$
2B	2·30	0·46	0·118	371·2	$3{\cdot}24 \,.\, 10^5$
2C	2·30	0·60	0·178	306·1	$1{\cdot}18 \,.\, 10^5$
2D	2·30	0·75	0·236	244·9	$5{\cdot}35 \,.\, 10^4$
2E	2·30	1·00	0·296	215·8	$3{\cdot}01 \,.\, 10^4$
2F	2·30	1·33	0·343	210·3	$2{\cdot}18 \,.\, 10^4$
2G	2·30	1·50	0·357	211·0	$2{\cdot}01 \,.\, 10^4$
3	2·00	1·00	0·286	231·0	$2{\cdot}26 \,.\, 10^4$
4	1·59	0·93	0·146	224·0	$4{\cdot}23 \,.\, 10^4$

For the creeping flow region we further computed the mean values $\overline{\mathrm{Kv}}$ for all experimental configurations. These values have been on the hand summarized in Table II and, on the other hand, presented as $\mathrm{Kv} = f(s/d)$ and $\mathrm{Kv} = f(D/d)$ dependences shown in Fig. 9. Table II gives also values of the energy criterion, E_c^*, computed from Eq. (*10*). The values of the product (PoRe) were taken from the first part of this work[1]. The values of E_c^* express the dimensionless energy consumption for recirculation of liquid and hence decrease with increasing lead of the screw (at constant D/d). The dependence of E_c^* on the D/d ratio expresses the minimal E_c^*, an optimum for $D/d \doteq 2$.

DISCUSSION

The limits of the creeping flow regime estimated in this work ($\mathrm{Re} < 15$) agree relatively well with the results of the power input criteria[1].

The found magnitude of the criterion $\overline{\mathrm{Kv}}$ are markedly dependent on the geometrical simplexes of the system. This is due to the role of both the pumping ability of the screw and the pressure drop within the system as a whole. The simplex s/d affects primarily the pumping capacity of the screw[9] while the simplex D/d, on the

TABLE III

A comparison of the flow rate criterion under the creeping flow regime

References	D/d	s/d	h_v/d	D'/d	Kv
Nagata[6]	2·23	0·67	2·00	1·11	0·237
This work	2·30	0·60	1·50	1·10	0·178
This work	2·30	0·75	1·50	1·10	0·236
Sýkora[5]	1·59	0·91	2·07	1·08	0·223
This work	1·59	0·93	1·50	1·10	0·146
Chavan[3]	2·25	1·00	2·25	1·05	0·178
Seichter[7]	2·30	1·00	1·50	1·10	0·326
This work	2·30	1·00	1·50	1·10	0·296
Carley[9]	—[a]	1·00	1·50	1·10	0·382
Rieger[2]	—[b]	1·00	3·00	1·02	0·384
This work	3·37	1·00	1·50	1·10	0·368

[a] Computed value for the drag flow in the screw ($\Delta p = 0$). [b] Experimental value for the drag flow in the screw ($\Delta p = 0$).

contrary, the magnitude of pressure drop due to recirculation of the highly viscous liquid in the mixing equipment[1].

The magnitude of the flow rate criterion, Kv, from this work has been compared in Table III with analogous data from the literature. Analysis of selected Kv data from Table III shows that our results fall within the limits of experimental values found by other authors. Data measured by the "flow follower" method[6,7] yield higher values owing to the tendency of the tracer particles to move in regions of the low pressure gradients[2]. Values obtained by volumetric measurements of the flow through the mixing vessel fall bellow[3] as well as above those from this work. The best agreement has been found for the arrangement with very low pressure drop (*i.e.* $D/d = 3{\cdot}37$) in comparison with the pressure-free drag flow[2,9].

A comparison of the effectiveness of individual arrangements in terms of the criterion E_c^* indicates that the screws with a low lead, $s/d < 0{\cdot}75$, are unsuitable for mixing highly viscous liquids. Use of screws with a high lead, on the other hand, is limited by the difficulties associated with their manufacture. The found optimum for the ratio $D/d \doteq 2$ then fully agrees with the results in other works[7,16].

LIST OF SYMBOLS

D	internal vessel diameter
D'	internal diameter of draught tube
d	diameter of screw impeller
d_0	diameter of screw core
H	height of liquid in vessel
H_2	height of impeller above bottom
h_v	height of impeller
k	constant, Eq. (*4*)
n	frequency of revolution
Δp	pressure drop in recirculation loop of the mixing system
R_0	electric resistance of thermistor under free convection
ΔR	resistance increase of thermistor due to forced convection
r	radical coordinate
r_2	radius of the outer (static) cylinder
s	lead of the screw
$\dot{V}$	pumping capacity of the screw; volume flow rate in vessel
v_z	axial velocity component
v_φ	tangential velocity component
λ	cylinder diameter ratio
μ	dynamic viscosity
ϱ	density
τ_c	mean time of circulation
τ_H	time of homogenation
$E_{c,H}^*$	energy criterion, see Eq. (*10*) or (*7*)
Kt	pressure criterion, see Eq. (*5*)
Kv	flow rate criterion, see Eq. (*3*)

Re Reynolds criterion for mixing (Re $= nd^2\varrho\mu^{-1}$)
* dimensionless value normalized by parameters n and d
— mean value

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